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REFRACTION PHENOMENA AFFECTING CEILOMETER OBSERVATIONS

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ABSTRACT

A typical ceilometer record during rainfall is examined and certain regular and recurring oscillations beginning uniformly at an angle of elevation of 48° and decreasing gradually with elevation are noted. These features are explained physically as "rainbow" effects. The angle of elevation of maximum oscillation is shown to be about 49° regardless of base-line length; the tapering off above is discussed. Possible "secondary rainbow" effects at lower elevation angles are mentioned. Finally, methods of differentiating between the rainbow effects and cloud layers are suggested.

INTRODUCTION

The standard Weather Bureau ceilometer usually gives an excellent record of the important meteorological element, cloud height [1]. However, with the onset of rain the record becomes confused by several peculiarities which are not easily explained at first glance. The chief characteristics of the record during a rainstorm, as indicated in figure 1, appear to be an oscillation which begins rather sharply at an angle of elevation of about 48° and which continues with decreasing intensity as the angle increases, and a second but intense oscillation at about the 80° elevation. These features of this particular record (fig. 1) are not unique, but are common to situations in which rain is falling from middle clouds in the absence of lower clouds. Variations are noticed in the height of the upper level at which the oscillations cease, but in nearly all cases the oscillation begins near 48°.

It has been noted that this "spot" just above the 48° elevation occurs at various stations throughout the United States.[2] The persistency with which this spot tends to occur at this angular elevation regardless of the base line would seem to indicate that the causes may be due to a raindrop refraction phenomenon rather than a diffused spot on a cloud base. In fact, since the ceilometer beam will not penetrate an appreciable thickness of cloud, it must be concluded that there are practically no clouds in the beam until it is extinguished at the highest level.

The aim of this paper is to study the ceilometer beam from a standpoint of atmospheric optics for the purpose of more accurately interpreting the record. More fully stated, the objectives of the paper are:

1. To demonstrate that when rain is falling, any spot showing in the ceilometer beam at (or very near) 49° should be regarded with caution in determining the height of the cloud base.
2. To present an explanation of other commonly observed oscillations in the ceilometer record during periods of rain. These consist mainly of:
   a. Attenuation of the spot with increasing elevation above 49°.
   b. Recurring oscillations near 36°.
   c. Recurring oscillations near the ground.
3. To develop criteria for deciding whether a spot is a reflection from a true cloud layer or merely a "rainbow" effect of the ceilometer beam shining through the raindrops.

PHYSICAL EXPLANATION OF SPOT NEAR 49°

The physical explanation of the refraction and reflection of sunlight by raindrops to cause the rainbow has long been known [2]. The light from the ceilometer projector

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[2] Mr. Ray Granger of the Oakland observing staff observed these spots at various stations. To him goes the credit for first pointing out such a spot to the writer and giving an indication of its true nature. Also as a direct result of Mr. Granger's alertness, the Weather Bureau Central Office issued special instructions to all ceilometer stations, pointing out the existence of this phenomenon and warning that it must not be confused with an actual cloud base. This phenomenon has been verified several times at the Air Force-Navy-Civil Landing Aids Experiment Station, Arcata, Calif., from simultaneous records by two ceilometers with different base lines (see L. A. E. S., *Final Reports, 1949*, Arcata, Calif., p. 27).

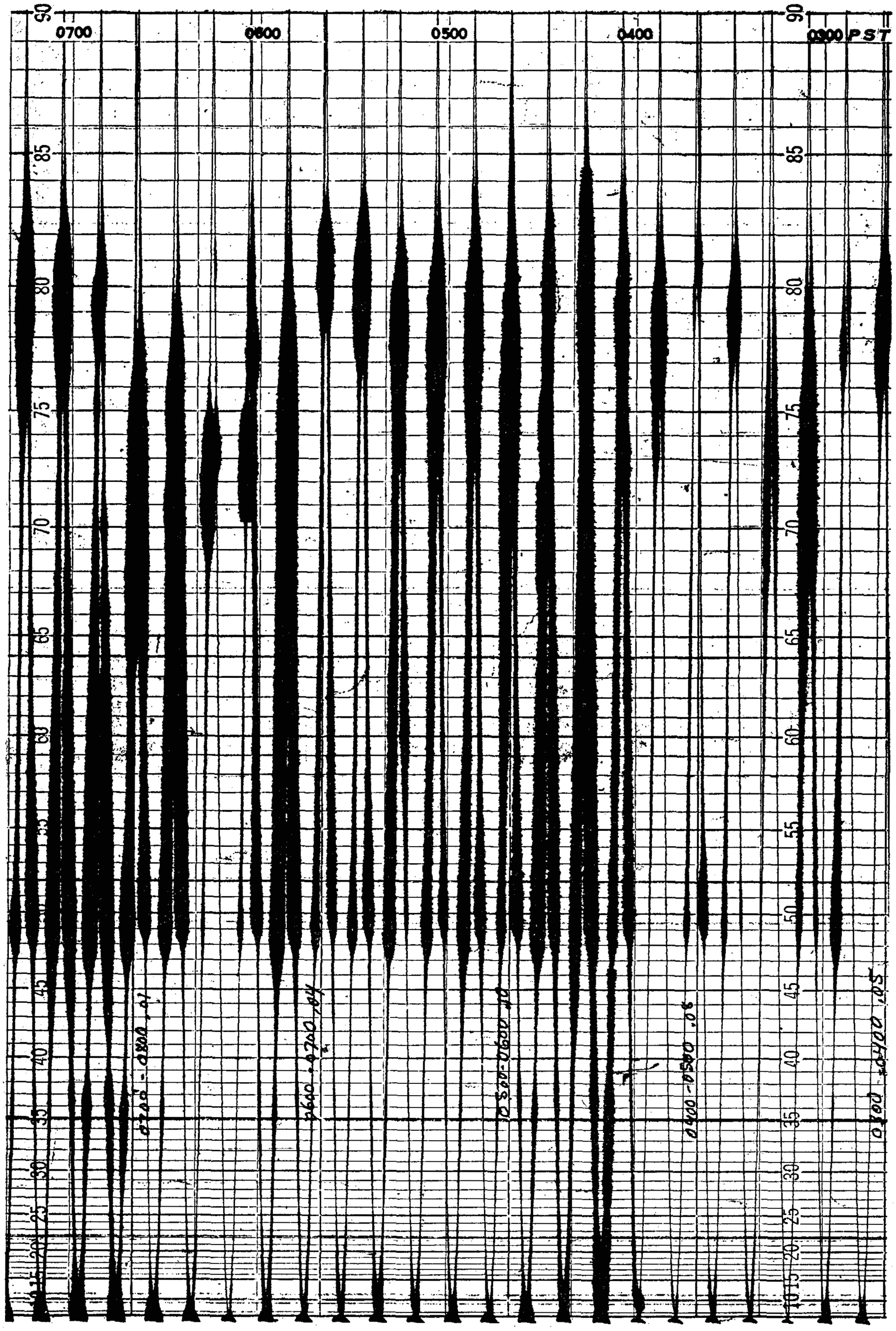

FIGURE 1.—A copy of a ceilometer record during a rain period. Oakland, Calif., December 18, 1949, 0300 to 0700 PST.

shining through rain presents a situation similar to that which results in the rainbow except that the beam is vertical and of very limited extent, and consists of but one color. In the following treatment, the problem of refraction by raindrops has been modified to take these factors into account.

SOLUTION OF THE REFRACTION PROBLEM

In the investigation of just how it is possible for a nearly monochromatic mercury vapor light refracted by raindrops aloft to produce bright areas within the beam, which result in the oscillations observed in the ceilometer record, three assumptions are made: (1) the freely falling drops are spherical; (2) the light from the mercury vapor projector beam is monochromatic with a value [3] of 5003Å; (3) the light rays from the ceilometer projector are parallel.

In figure 2, the ray from the mercury vapor light source S enters the drop at A at an arbitrary angle of incidence i, and is refracted at the angle of refraction r within the drop. The ray is then reflected at B by the upper spherical surface of the drop and then refracted again as it emerges at C at the angle of emergence e. The problem is to get an expression for the angle of deviation D, which is the total change of direction which the light ray suffers between entrance and emergence from the drop.

First, it will be useful to find from the geometry of figure 2 the relation of r to r' and of i to e. Angle OBA equals angle OBC because the radius OB is also the axis of reflection of the spherical surface in the region of B. Hence, the two isosceles triangles AOB and BOC are congruent and $r'=r$. Now, since $\sin i=\mu \sin r$, and $\sin e=\mu \sin r'$ (where μ is the index of refraction), $i=e$; the angle of incidence equals the angle of emergence.

With these results, an expression for D, the total deviation of the light ray, may be found in terms of i and r. In going from S to A to B, the ray is deviated clockwise through an angle $(i-r)$. Since angle $ABC=2r$ (see above), in traversing the inside of the drop from A to B to C the ray is rotated clockwise about B through an angle $(180°-2r)$. Finally it is rotated clockwise through the angle $(e-r')$ in going from B to C to E. Summing these rotations gives a total deviation $D=(i-r)+(180°-2r)+(e-r')$. Since $(e-r')=(i-r)$,

$$D=180°+2i-4r \tag{1}$$

If parallel rays are incident on a sphere, the angles of incidence vary between 0° (for the ray traversing the center) and 90° (for the rays tangential at the edges). For any arbitrary angles of incidence i, corresponding values of r may be computed and substituted in equation (1) to determine D, the final deviation. A series of computations using different values for angle i yields the curve of D against i given in figure 3. This curve was computed for index of refraction $\mu=1.336$ when wavelength $\lambda=5003$Å, temperature $T=20°$ C., and pressure $p=1$ atmosphere (see [3]).

To find an expression for the minimum point of the curve, differentiate equation (1) and set $dD=0$. This gives $di=2dr$; but $\sin i=\mu \sin r$, and $\cos i \cdot di=\mu \cos r \cdot dr$. Thus, $\mu \cos r=2 \cos i$, or

$$\cos i=\sqrt{\frac{\mu^2-1}{3}} \tag{2}$$

when D is a minimum. Setting $\mu=1.336$ in (2) gives $\cos i=0.5116$ and $i=59°14'$.

For this value of i, the angle of refraction equals $40°2'$. Substituting $i=59°14'$ and $r=40°2'$ into (1) gives a minimum value of D equal to $138°\ 20'$ for the beam from the

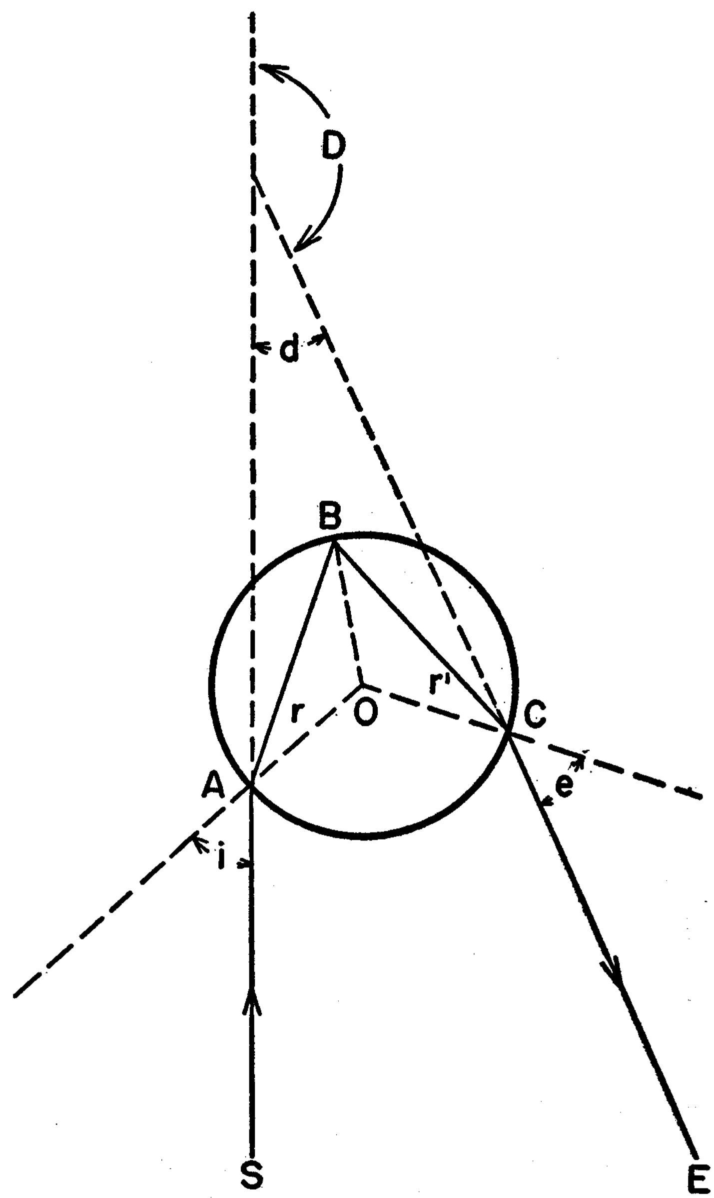

FIGURE 2.—Diagram showing the path which a vertical ray of light takes in passing through a raindrop near minimum deviation in which one internal reflection takes place. As shown by this diagram the angle of elevation equals $90°-d=D-90°$ (where $D+d=180°$).

[3] The mean value of the two principal mercury lines at wavelengths 4358Å and 5461Å is 4910Å; the nearest wavelength λ for which Dorsey [3] gives a good value of the index of refraction μ is $\lambda=5003$Å.

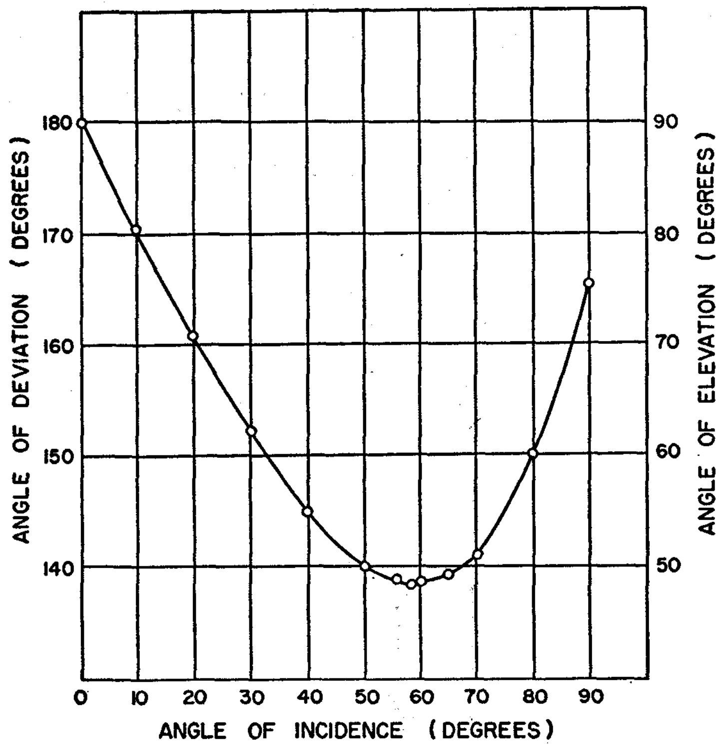

FIGURE 3.—This graph shows the relationship between the angle of incidence and the angle of deviation for a monochromatic ray of light passing through a raindrop, as obtained from the equation $D=180°+2i-4r$, with $\sin r=\frac{\sin i}{\mu}$, $\mu=1.336$. The angle of elevation for a corresponding angle of deviation has been entered along the right side of the graph.

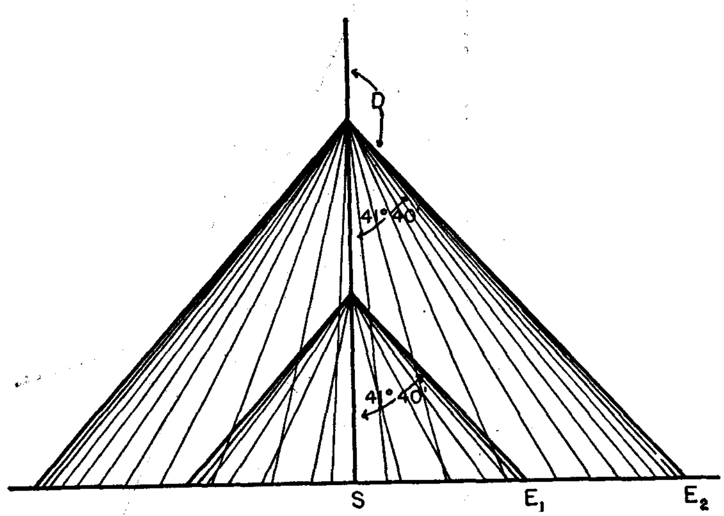

FIGURE 4.—A schematic diagram showing a right circular cone resulting from a beam of light shining upward through raindrops. The returning rays of light are all confined within a cone of which half the angle is 41°40′ due to the fact that no ray is deviated less than 138°20′. Receivers at E_1 and E_2 both would detect the bright spot at the same angle of elevation.

mercury vapor light. Normal rays are deviated 180° by reflection and tangential rays are deviated about 166°. Since no ray is deviated less than 138° 20′, it follows that the rays emerging from the individual drop are all contained within a right circular cone, of which half the vertical angle, d, equals 41° 40′ as illustrated in figure 4.

FORMATION OF SPOT NEAR 49°

Near the point of minimum deviation, as shown by figure 3, a relatively large change in the angle of incidence results in only a small change in the angle of deviation, D. Consequently, there is a tendency for light rays to be congested near the minimum angle of deviation of 138°20′, which corresponds to the angle of elevation of 48°20′ (the elevation angle$=D-90°=90°-d$). However, the greatest intensity of illumination in the beam occurs at a slightly larger deviation angle [2], thus causing the brightest area to appear just above the angle of elevation of 48°20′.

While the curve in figure 3 is constructed for a monochromatic light beam with an index of refraction of 1.336, a similar curve could be constructed for every individual wave length in the spectrum, the index of refraction, μ, varying with the wave length. For red light $\lambda=7065$Å, $\mu=1.330$; for violet light $\lambda=4102$Å, $\mu=1.342$. If curves like the above are constructed for these wave lengths, half of the vertical angle, d, of the cone of emergent rays is 42.1° for red light and 40.2° for violet. If the ceilometer beam were made up of all wave lengths, a very short section of a rainbow would be visible at the spot between 47.9° and 49.8° elevation with the red at the lower and the violet at the upper position. From this it may be seen that the spot observed at about the 49° elevation during rainfall may be considered as a short section of a "monochromatic rainbow" lying in a horizontal plane above the observer.

DEPARTURES FROM THEORY

It should be understood that the value of $d=41°40'$ given above is only a theoretical value dependent upon the initial assumptions. However, we would expect to find the observed value very near that indicated by theory as only small errors would be introduced into the value of d by departures from the initial assumptions. Slight variations in the value of d, with corresponding changes in the angle of elevation, would likely be caused by the following factors: (1) the size of the spot would be increased slightly by the dispersive effects of the nonspherical drops; (2) μ for distilled water varies slightly with the temperature (for $\lambda=5016$Å, $\mu_{10°C}=1.337070$, $\mu_{20°C}=1.3363453$, $\mu_{30°C}=1.335289$, a variation in the fourth figure only); (3) the combination of the two wave lengths 4358Å and 5461Å will result in more spreading of the spot than would be present in purely monochromatic light; (4) the lack of strict parallelism of the rays would introduce only a slight variation in d; (5) slight errors may be present in the ceilometer, thus affecting the measurement of the angle of elevation.

The first four (physical) factors, acting accumulatively, should not give a departure of more than one degree from the theoretical value. The fifth (instrumental) effect is purely a local problem. A consistent occurrence of these spots at 45°, for example, should lead one to investigate whether or not the light has been jarred out of normal alignment.

PHYSICAL EXPLANATION OF OTHER FEATURES

The discussion thus far offers a satisfactory explanation for the formation of the main spot in the ceilometer beam at about 49°. Additional features noted on the ceilometer record and shown schematically in figure 5 remain to be explained. These features may be listed as follows: (1) The gradual weakening of the intensity of the oscillation between 49° and about 76°; (2) The apparent tendency for a weak spot to form at about 36°; (3) The existence of the oscillation near the surface, which occasionally extends upward until it merges with the spot at 49°.

DECREASE IN INTENSITY ABOVE 48°

The diminution of the oscillation with increasing angle above the main spot, A, as indicated in figure 5, appears to be nearly inversely proportional to the angle of elevation. A qualitative explanation of the decrease in intensity may be obtained through a consideration of the effectiveness of the raindrop in returning the light ray for varying angles of incidence and the increasing loss of light by scattering for the longer path lengths.

Each raindrop in the light beam deflects light to the receiver either by internal or external reflection, or both. Only those drops from the angle of elevation of 48°20′ upward to the cloud base will return light by one internal reflection in addition to that reflected at the surface. As previously mentioned, and as indicated in figure 3, there is a packing of the internally reflected rays near the angle of minimum deviation. For angles of deviation greater than this minimum value, but less than 166° (see fig. 3), there are two rays with different angles of incidence which have the same deviation angle. For example, rays incident at angles of about 20° and 87° suffer the same total deviation. As the steepness of both arms of the curve increases, the congestion of the light rays diminishes for angles of incidence farther and farther away from the value of 59°14′. As a consequence, we should expect the light from the beam to decrease in intensity with elevation above the main spot.

The greater path lengths for portions of the beam at the higher elevations would also tend to diminish the amount of light returned, due to the increased scattering. Other factors, such as the variation of the amount of light reflected for varying angles of incidence, would have to be considered to obtain a complete solution to the problem. The decrease in the intensity of the light for angles of deviation greater than the minimum value has its counterpart in the well-known fact that the area beneath the rainbow is brighter than that above it.

A classical solution to the problem of the intensity of the wave front and its variation with angular distance from the ray of minimum deviation was first obtained by Airy [4] and a later solution was given by Mascart [5] and Pernter-Exner [6] which is outlined by Humphreys [2]. The solution given by Humphreys shows that the maximum intensity does not coincide with the angle of minimum deviation, but occurs a short distance from it. Beyond the first maximum, which is the brightest, are succeeding maxima which decrease in intensity and angular intervals at a decreasing rate.

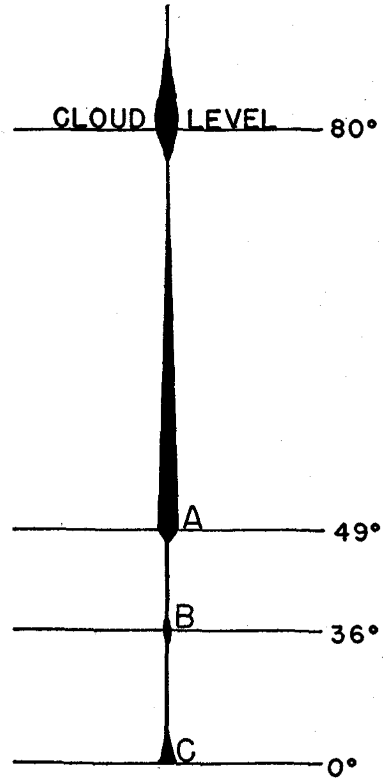

FIGURE 5.—A schematic representation of the most common oscillations recorded by the ceilometer during periods of rainfall. A, main spot; B, secondary spot; C, surface fog and haze.

FORMATION OF A WEAK SPOT NEAR 36°

During periods of heavy rainfall, a weak spot frequently appears near the 36° angle of elevation. So far in the discussion no mention has been made of the possibility of a second reflection within the raindrop causing a "secondary spot" comparable to the secondary rainbow. The intensity of light from a secondary rainbow is usually very much less than that from the primary bow and shows wide variation in intensity depending upon the type of rain. Accordingly, a double reflection within the raindrops in the ceilometer beam would not be expected to give such a consistent spot as that for a single reflection or to be of comparable intensity.

The total deviation and the angle of minimum deviation may be calculated for the double reflection in much the same manner as that for a single reflection. From Humphreys [2] we get

$$D=360°+2i-6r \text{ and } \cos i=\sqrt{\frac{\mu^2-1}{8}}$$

Proceeding with the calculations as before yields $i=71°\ 46'$, $r=45°\ 19'$, and $D=231°\ 38'$. In this case, $d=D-180°=51°\ 38'$. Hence, the spot should appear near the angle of elevation of 38° 22′ (90°−51° 38′).

As with the primary spot, the greatest illumination would not be expected to be at the position of the minimum deviation, but rather displaced a short distance toward the direction of increasing deviation. It was seen that for the primary spot, this direction was upward; similar considerations would indicate that for the secondary spot the direction would be downward. Thus, qualitatively, we should expect the secondary spot as picked up by the ceilometer to be below the elevation angle of 38° 22′. This factor, together with the introduction of the same errors affecting the primary spot, is probably sufficient to account for the appearance of the spot between 36° and 37°, as indicated at *B* in the schematic diagram in figure 5.

OSCILLATIONS AT VERY LOW ELEVATIONS

The increase in oscillation near the surface during periods of rainfall may be due largely to the formation of fog or haze in this region as a result of the precipitation. There is another factor of importance, however, and that is the direct reflection from the surface of the raindrops.

As may be seen from figure 6, the observer receives reflected light from a whole column of drops in the light beam, the angle of reflection decreasing with the height of the drop. The greatest amount of reflected light would be from a ray reflected tangential to the surface of a drop infinitely far below the observer. Since the observer is not able to see drops lower than the ceiling projector, the amount of reflected light would be greatest for a 0° angle of elevation and would decrease with an increasing angle.

During periods of heavy rain a continuous oscillation is occasionally recorded between *B* and *C* in figure 5. This type of oscillation is probably caused to a large extent by the external reflection from the drops. The more common oscillation occurring between 0° and 15° elevation (see fig. 5) more likely results from haze and fog forming in this region.

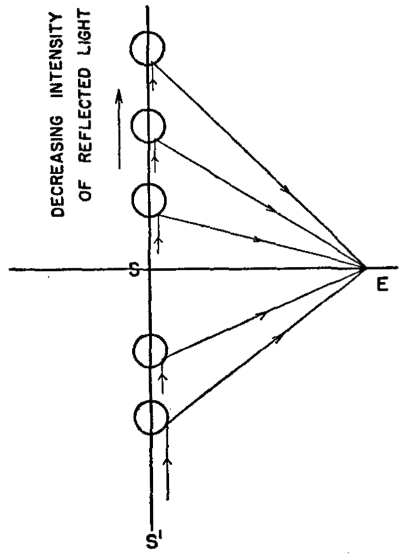

FIGURE 6.—Diagram indicating the decreasing angle of incidence with increasing elevation angle of the raindrop; the decreasing angle of incidence results in a diminishing of the light returned to the receiver at *E*.

PRACTICAL APPLICATIONS

Now that the various ways in which the ceiling light and rain drops can produce oscillations in the receiver have been investigated, means of differentiating between refraction phenomena and actual cloud layers will be discussed.

At night the problem is simplified by the fact that the spot is visible. All the observer need do is walk a short distance toward the ceiling light; if the elevation angle of the spot remains about 49°, the light is not being reflected from a cloud layer.

In the daytime other methods must be used. If the following points are kept in mind, the observer should be able to recognize the refraction phenomena from the ceilogram alone; (1) the spots occur regularly near 49° elevation as long as rain continues; (2) the intensity of the oscillation should follow roughly the intensity of rainfall; (3) all of the refraction phenomena described allow further penetration of the beam until a true cloud layer is reached at a higher elevation (e. g., at about 80° as shown in fig. 5). Ceiling balloons and frequent checks with incoming and outgoing aircraft may be used if doubt still exists.

ACKNOWLEDGMENT

The writer wishes to express his gratitude for the valuable help rendered by Mr. Donald L. Jorgensen in preparing the manuscript. Mr. Jorgensen also pointed out the importance of the gradually decreasing oscillations above 49° and the probable cause.

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